Design Method for Geogrid-Reinforced Unpaved Roads. II. Calibration and Applications

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Abstract: A theoretically based base course thickness design method for unpaved roads was developed in the companion paper. This paper presents a calibration of the design method using data from field wheel load tests and laboratory cyclic plate loading tests on unreinforced and reinforced base courses. The constants in the design method are determined during the calibration. The calibrated design method is used for analyzing the test data through three case studies. In addition, the design procedures and a design example are provided in this paper to demonstrate the use of the design method.

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Introduction

Recent field and laboratory test data (Fannin and Sigurdsson 1996; Knapton and Austin 1996; Gabr 2001; Tingle and Webster 2003) show the need for an improved design method for geogrid-reinforced unpaved roads. Importantly, these studies provide data needed for calibration and verification of the method. Considering distribution of stress, strength of base course material, interlock between geosynthetic and base course material, geosynthetic stiffness, traffic volume, wheel loads, tire pressure, subgrade strength, rut depth, and influence of the presence of a reinforcing geosynthetic (geotextile or geogrid) on the failure mode of the unpaved road or area, the writers developed a theoretically based base course thickness design method for unpaved roads presented in the companion paper (Giroud and Han 2004). The required base course thickness for unpaved roads can be calculated using the following equation:

\[ h = \frac{1.26 + (0.96 - 1.46f^2)\left(\frac{r}{h}\right)^{1.5}}{f_E} \log N - \frac{p}{\pi r^2 m N_c c_u} - 1 \frac{r}{r} \]

where \( h \) = required base course thickness (m); \( P \) = wheel load (kN); \( N \) = number of passes of axle; \( J \) = aperture stability modulus of geogrid (with \( J = 0 \) for unreinforced and geotextile-reinforced unpaved roads); and \( r \) = radius of equivalent tire contact area, which can be determined by Eq. (3) in the companion paper, i.e.

\[ r = \sqrt{\frac{p}{\pi p}} \] (2)

\[ p = \text{tire contact pressure. The undrained cohesion of subgrade soil, } c_u, \text{ can be determined using Eq. (5) in the companion paper, i.e.} \]

\[ c_u = f_C \text{CBR}_{sg} \] (3)

where \( \text{CBR}_{sg} \) = subgrade California bearing ratio; and \( f_C = \text{factor equal to 30 kPa.} \)

\[ m = \left(\frac{s}{f_s}\right)^1 \left[1 - \xi \exp\left[-\omega \left(\frac{r}{h}\right)^v\right]\right] \] (4)

where \( s = \text{rut depth; and } f_s = \text{factor equal to 75 mm rut depth. } \xi \), \( \omega \), and \( n \) = unknown parameters that will be determined later in this paper, when the method is calibrated with experimental data.

The modulus ratio factor, \( f_E \), is expressed as Eq. (22) in the companion paper, i.e.

\[ f_E = 1 + 0.204(R_E - 1) \] (5)

where \( R_E = \text{limited modulus ratio defined by} \)

\[ R_E = \min\left(\frac{E_{bc}}{E_{sg}}\right) \leq 5.0 \] (6)

where \( E_{bc} = \text{base course California bearing ratio; } \text{CBR}_{sg} = \text{subgrade soil California bearing ratio; } E_{sg} = \text{base course resilient modulus; and } E_{sg} = \text{subgrade soil resilient modulus.} \)

Based on the discussions in the companion paper, the bearing capacity factor should be used: \( N_c = 3.14 \) for unreinforced unpaved roads, \( N_c = 5.14 \) for geotextile-reinforced unpaved roads, and \( N_c = 5.71 \) for geogrid-reinforced unpaved roads.

Calibration of Design Equations

Equation Calibration Using Field Data

Introduction

The numerical values of the constants in the numerator of Eq. (1) were obtained from laboratory cyclic plate load tests by Gabr...
(2001). Such laboratory tests do not accurately represent field conditions. Calibration of the equation using field data is necessary. The unknown constants ($\xi$, $\omega$, and $n$) that define the bearing capacity mobilization coefficient [see Eq. (4)] are also determined in the calibration process.

For calibration, Eq. (1) can be written as follows:

$$h^* = a + (b - d J^2) \left[ \frac{r}{h} \right]^{1.5} \log N$$

(7)

where $a = 1.26$, $b = 0.96$, $d = 1.46$, and $h^*$ is the normalized base course thickness defined by

$$h^* = h f_{\xi} \left[ \frac{P}{\pi r^2 m N f_s CBR_{sG}} - 1 \right]^{-1}$$

(8)

It should be noted that, in Eq. (8), $c_u$ has been replaced by $f_s CBR_{sG}$, in accordance with Eq. (3). This was done because Hammitt’s data used for calibration were provided as a function of the $CBR$ of the subgrade soil, not as a function of undrained cohesion.

** Calibration of Constants $a$ and $b**

Calibration of Eq. (7) was done using field data from unreinforced unpaved roads collected by Hammitt (1970). These data were selected for calibration because they are the most complete set of field data on unpaved road performance to the writers’ knowledge. Since there was no reinforcement in Hammitt’s tests, $J = 0$ in Eq. (7), and only the constants $a$ and $b$ could be determined in the calibration process. Calibration of the constant $d$ requires a special approach, as discussed later. Limited field data on reinforced unpaved roads will be used in the “Case Studies” section to confirm the validity of the method.

As noted in the companion paper, the method developed in this study is based on the assumption that subgrade failure is the controlling factor. Therefore any test data presented by Hammitt (1970) that could be attributed to failure within the base course rather than the subgrade was not considered for method calibration purposes. The methodology used to identify those of Hammitt’s tests that could be attributed to base course failure is as follows. Hammitt (1970) presents a design chart to estimate the number of axle passes that can be carried by an unsurfaced soil (i.e., a soil not covered with a base course) based on wheel load, tire pressure, and $CBR$ of the soil. The wheel-carrying capacity of a base course itself can be estimated using this chart with the base course $CBR$ instead of the soil $CBR$. The wheel-carrying capacity of the base course for all of Hammitt’s tests was thus evaluated, and the test results that correspond to a wheel load that equaled or exceeded this estimated wheel-carrying capacity were not used. Using the remaining test data from Hammitt (1970), the two constants, $a$ and $b$, can be determined by a regression analysis using the equation for the unreinforced case. As shown in Fig. 1, the regression analysis performed with Eq. (7) gives

$$a_{\text{field}} = 0.866 \quad \text{and} \quad b_{\text{field}} = 0.663$$

(9)

where $a_{\text{field}}$ = value of $a$ obtained from calibration with field test data; and $b_{\text{field}}$ = value of $b$ obtained from calibration with field test data.

The calibration of the constants $a$ and $b$ will be further refined in a subsequent section.

** Calibration of Bearing Capacity Mobilization Factor**

The constant $\xi$ must be close to 1.0 because $m$ must be small when $r/h$ is small. To avoid numerical instability due to the bearing capacity mobilization coefficient approaching zero at small values of the $r/h$ ratio, the constant $\xi$ was set at 0.9 rather than 1.0. Then, the constants $\omega$ and $n$ were determined as explained below.

Calibration using field wheel load tests of the method that was developed based on cyclic plate loading tests requires a relationship between these two types of tests. It is assumed that the plots of $1/\tan \alpha$ versus $\log N$ for the two loading conditions are parallel. To establish this parallel relationship, the values of $\omega$ and $n$ were varied until parallel plots were achieved for laboratory and field data. The set of $\omega$ and $n$ that gave the highest value for the coefficient of multiple correlation, $R^2$, was selected. The values thus obtained for the constants $\omega$ and $n$ were 1.0 and 2.0. No theoretical explanation was found for the fact that these two values are very simple.

As a result of the calibration process described above, Eq. (4) becomes

$$m = \left( \frac{S}{f_s} \right) \left[ 1 - 0.9 \exp \left( -\left( \frac{r}{h} \right)^{2} \right) \right]$$

(10)

The range of variation of $m$ is discussed later.

** Calibration of Constant $d$**

The field tests by Hammitt (1970) are considered more representative of actual conditions in unpaved roads than the laboratory tests by Gabr (2001), but they do not enable calibration of the constant $d$ of Eq. (7). To determine the constant $d$, the following approach is used:

- The ratios between the values of $a_{\text{field}}$ derived from field tests and the value of $a_{\text{lab}}$ derived from laboratory tests and between $b_{\text{field}}$ and $b_{\text{lab}}$ are calculated.
- These two ratios are averaged.
- The averaged ratio is then used to calculate a new value of $d$ from the value of $d_{\text{lab}}$ derived from laboratory tests. The value of “$d$” thus calculated is then considered representative of field conditions.
- The ratios of values for field and laboratory conditions are as follows:

$$\frac{a_{\text{field}}}{a_{\text{lab}}} = 0.866 / 1.26 = 0.687$$

(11)

$$\frac{b_{\text{field}}}{b_{\text{lab}}} = 0.663 / 0.96 = 0.691$$

(12)

Fig. 1. Determination of constants, $a$ and $b$
Note that the ratios are close to equal. The average ratio of 0.689 is applied to $d$:

$$d_{\text{fieldback}} = 0.689(-1.46) = -1.006$$ (13)

where $d_{\text{fieldback}}$ = value of $d$ back-calculated to be representative of field conditions.

**Further Calibration of Constants $a$ and $b$**

For the sake of consistency with the calibration of $d$, back-calculated values of $a_{\text{field}}$ and $b_{\text{field}}$ are obtained as follows:

$$a_{\text{fieldback}} = 0.689 \times 1.26 = 0.868$$ (14)

$$b_{\text{fieldback}} = 0.689 \times 0.96 = 0.661$$ (15)

where $a_{\text{fieldback}}$ = value of $a$ back-calculated to be representative of field conditions; and $b_{\text{fieldback}}$ = value of $b$ back-calculated to be representative of field conditions.

It is considered that $a_{\text{fieldback}}$, $b_{\text{fieldback}}$, and $d_{\text{fieldback}}$ are the best calibrated values of $a$, $b$, and $d$.

**Design Equations**

Inserting the calibrated values $a_{\text{fieldback}}$, $b_{\text{fieldback}}$, and $d_{\text{fieldback}}$ into Eq. (7), and combining Eqs. (7) and (8), gives

$$h = \frac{0.868 + (0.661 - 1.006J^2) \left( \frac{r}{h} \right)^{1.5} \log N}{f_E}$$

$$\times \left[ \sqrt{\frac{P}{\pi r^2}} \left( 1 - 0.9 \exp \left( - \frac{r}{h} \right) \right)^{N_c CBR_{sg}} - 1 \right] r$$ (16)

where the modulus ratio factor, $f_E$, is defined by Eq. (5) and the bearing capacity mobilization factor, $m$, is defined by Eq. (10).

Combining Eqs. (5), (10), and (16) gives

$$h = \frac{0.868 + (0.661 - 1.006J^2) \left( \frac{r}{h} \right)^{1.5} \log N}{1 + 0.204[R_E - 1]}$$

$$\times \left[ \sqrt{\frac{P}{\pi r^2}} \left( 1 - 0.9 \exp \left( - \frac{r}{h} \right) \right)^{N_c CBR_{sg}} - 1 \right] r$$ (17)

where $h$ = required base course thickness (m); $J$ = geogrid aperture stability modulus (m $N^2$); $N$ = number of axle passages; $P$ = wheel load (kN); $r$ = radius of the equivalent tire contact area (m); $R_E$ = limited modulus ratio of base course to subgrade soil [given by Eq. (6)]; $s$ = allowable rut depth (mm); $f_S$ = factor equal to 75 mm; $N_c$ = bearing capacity factor; $f_C$ = factor equal to 30 kPa [see Eq. (3)]; and $CBR_{sg}$ = $CBR$ of the subgrade soil. According to Eq. (2), $P/(\pi r^2)$ is equal to the tire contact pressure, which is close to the tire pressure.

It should be noted that Eqs. (16) and (17) are expressed as a function of the $CBR$ of the subgrade soil. If the strength of the subgrade soil is defined using the undrained cohesion, the following equations derived by combining Eq. (3) with Eqs. (16) and (17), respectively, should be used:

$$h = \frac{0.868 + (0.661 - 1.006J^2) \left( \frac{r}{h} \right)^{1.5} \log N}{1 + 0.204[R_E - 1]}$$ (18)

$$h = \frac{0.868 + (0.661 - 1.006J^2) \left( \frac{r}{h} \right)^{1.5} \log N}{1 + 0.204[R_E - 1]}$$ (19)

Strictly speaking, Eqs. (16) and (17) are more accurate than Eqs. (18) and (19), respectively, because calibration was done with equations derived from Eq. (3) and, therefore, expressed as a function of the $CBR$ of the subgrade. Therefore Eqs. (18) and (19) should be used only for cases where the relationship between the undrained cohesion and the $CBR$ of the subgrade soil is close to the relationship expressed by Eq. (3).

Eqs. (16) and (17) are equivalent and Eqs. (18) and (19) are equivalent. Because the required base course thickness, $h$, appears on both sides of Eqs. (16)–(19), iterations are required to determine $h$. It is important to note that Eqs. (16)–(19) are valid for both unreinforced and reinforced unpaved roads. For unreinforced unpaved roads, $J=0$ and $N_c=3.14$. For geotextile-reinforced unpaved roads, $J=0$ and $N_c=5.14$. For geogrid-reinforced unpaved roads, $J>0$ and $N_c=5.71$. Design charts were prepared using Eq. (17). These charts are presented in Fig. 2.

The thickness of an unreinforced unpaved road calculated using Eq. (16) or (17) is compared in Fig. 3 with the actual thickness reported by Hammitt (1970). The comparison shows that Eq. (16) or (17) gives a value of 0.73 for the coefficient of multiple correlation, $R^2$. A similar comparison using Hammitt’s data with his equation gives an $R^2$ value of only 0.29. Eq. (16) or (17) therefore provides a significantly improved prediction of performance compared to Hammitt’s equation.

The wheel load, $P$, can be calculated as a function of the base course thickness, using any of the following equations derived from Eqs. (16)–(19):

$$P = \pi r^2 m N_c f_C CBR_{sg}$$

$$\times \left[ 1 + 0.204(R_E - 1) \right]$$ (20)

$$P = \pi r^2 \left( \frac{s}{f_S} \right) \left[ 1 - 0.9 \exp \left( - \frac{r}{h} \right)^2 \right] N_c f_C CBR_{sg}$$

$$\times \left[ 1 + 0.204(R_E - 1) \right]$$ (21)
Allowable Rut Depth and Bearing Capacity Mobilization Factor

As mentioned in the companion paper, a typical allowable rut depth is 75 mm. However, the design engineer may select another allowable rut depth, such as 100 mm. The smaller the selected allowable rut depth, the greater the required base course thickness, all other parameters being equal. If the selected allowable rut depth is greater than 75 mm, it is necessary to check that the deflection at the base course/subgrade interface is less than 75 mm because it was assumed that this is the condition that governs the occurrence of limit equilibrium in the subgrade. This is achieved by checking that the bearing capacity mobilization coefficient, \( m \), is not greater than unity. If the bearing capacity mobilization coefficient, \( m \), is greater than unity, the base course thickness must be increased or a smaller allowable rut depth must be selected.

Another approach (which is equivalent to checking that \( m \) is not greater than unity) consists of checking that the base course thickness is greater than the minimum value, \( h_{\text{min}} \), given by the following equation derived from Eq. (10):

\[
P = 2 \pi F S \left( \frac{h}{f} \right) \left( 1 - 0.9 \exp \left( - \left( \frac{E}{h} \right)^2 \right) \right) N_c c_u \\
\times \left[ 1 + \frac{0.868 + (0.661 - 1.006 F^2) \left( \frac{E}{h} \right)^{1.5} \log N}{0.868 + (0.661 - 1.006 F^2) \left( \frac{E}{h} \right)^{1.5} \log N} \right]^{2}
\]

Eqs. (20) and (21) are equivalent and Eqs. (22) and (23) are equivalent. It is important to note that iterations are not needed to solve Eqs. (20)–(23).

Discussion

Allowable Rut Depth and Bearing Capacity Mobilization Factor

As mentioned in the companion paper, a typical allowable rut depth is 75 mm. However, the design engineer may select another allowable rut depth, such as 100 mm. The smaller the selected allowable rut depth, the greater the required base course thickness, all other parameters being equal (Fig. 2).

If the selected allowable rut depth is greater than 75 mm, it is necessary to check that the deflection at the base course/subgrade interface is less than 75 mm because it was assumed that this is the condition that governs the occurrence of limit equilibrium in the subgrade. This is achieved by checking that the bearing capacity mobilization coefficient, \( m \), is not greater than unity. If the bearing capacity mobilization coefficient, \( m \), is greater than unity, the base course thickness must be increased or a smaller allowable rut depth must be selected.

Another approach (which is equivalent to checking that \( m \) is not greater than unity) consists of checking that the base course thickness is greater than the minimum value, \( h_{\text{min}} \), given by the following equation derived from Eq. (10):
The symbol for the bearing capacity thus calculated is \( P_{h=0,\text{unreinforced}} \). The value of \( P_{h=0,\text{unreinforced}} \) is independent of the number of axle passes. Theoretically, no base course is needed in this case. However, a minimal base course thickness of 0.10 m is recommended, as indicated in the companion paper.

If Eq. (25) or (26) is used with \( N_c = 5.14 \) (for geotextile) or \( N_c = 5.71 \) (for geogrid), the bearing capacity thus calculated (\( P_{h=0,GTX} \) for the geotextile case or \( P_{h=0,GGD} \) for the geogrid case) is the wheel load that can theoretically be carried by the subgrade soil just covered with a geotextile or a geogrid (i.e., with no base course). The value of \( P_{h=0,GTX} \) or \( P_{h=0,GGD} \) is independent of the number of axle passes. In reality, the geotextile or the geogrid would deteriorate due to traffic and a minimal base course thickness of 0.10 m is recommended.

There are three cases where a negative value of the required base course thickness is rightfully obtained. A negative value is obtained if the required base course thickness, \( h \), is calculated using any of Eqs. (16)–(19): (1) for a wheel load smaller than \( P_{h=0,\text{unreinforced}} \) in the case of an unreinforced base; (2) for a wheel load smaller than \( P_{h=0,GTX} \) in the case of a geotextile-reinforced base; and (3) for a wheel load smaller than \( P_{h=0,GGD} \) in the case of a geogrid-reinforced base. In these cases, a minimal base course thickness of 0.10 m is recommended.

**Limitation of Method Presented in This Study**

As indicated earlier, the validity of the method presented in this paper is limited to rut depths ranging between 50 and 100 mm. More calibration work, based on more field data, would be required to extend the validity of the method to a broader range of rut depths. When this calibration work is done, it may appear that the assumption of proportionality between rut depth and bearing capacity mobilization factor should be replaced by another assumption.

Eq. (3) is valid only for a \( CBR \) of the subgrade soil less than 5.0 and the relationship between modulus and \( CBR \) of subgrade soil [Eq. (6) in the companion paper] is valid only for a \( CBR \) of the subgrade soil less than 10.0. Therefore the method presented in this study is valid only for subgrade soils with a \( CBR \) less than 5.0. This limitation does not significantly restrict the use of the method presented in this study because reinforced unpaved roads are generally constructed on soils with a \( CBR \) less than 3.

As indicated in the companion paper, a maximum limit of 5.0 is used in the method presented in this study for the ratio between the base course material modulus and the subgrade soil modulus. This modulus ratio limit would be increased if the base course material modulus were increased. This could be the case if the presence of a geogrid at the base/subgrade interface were to result in improved compaction of the base course aggregate. Improved compaction of the base course aggregate would have another benefit: it would increase the bearing capacity of the base course with respect to failure above the geogrid. This seems to be confirmed by a study by Wayne et al. (1998). However, this study is not conclusive because it was based on a footing under a static condition. Further studies would be needed to evaluate the ability of geogrid reinforcement to increase compaction of the base course aggregate and, therefore, increase the modulus of the base course aggregate and the bearing capacity of the base course.

From a mathematical standpoint, there is no limitation in the method presented in this study regarding the number of axle passes. In other words, Eqs. (16)–(19) (that give the required base course thickness) and Eqs. (20)–(23) (that give the wheel load that can be carried by a given base) can be used with any value of \( N \) from one to infinity. It should be noted that the maximum traffic

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\[ h_{\text{min}} = \frac{r}{\sqrt{\ln \left( \frac{0.9}{1 - \frac{f_s}{s}} \right)}} \]  
(24)

For an allowable rut depth \( s = 100 \text{ mm} \), Eq. (24) shows that the minimum value of the base course thickness is 0.884\( r \). If the equivalent wheel radius is 0.15 m (a typical value), the minimum value of the base course thickness is 0.13 m. From a practical standpoint, it may be considered that an allowable rut depth of 100 mm can be selected if the base course thickness is equal to or greater than the radius of the equivalent wheel load.

Fig. 4 gives the bearing capacity mobilization coefficient, \( m \), as a function of the \( h/r \) ratio and the value of the allowable rut depth, \( s \). This figure also shows the minimum base course thickness for \( s = 80, 90, 100 \text{ mm} \) (\( h_{\text{min}} = 0.612r, 0.770r, \) and 0.884\( r \), respectively). No allowable rut depth value smaller than 50 mm or greater than 100 mm is shown in Fig. 4 because the use of the design method with allowable rut depth smaller than 50 mm or greater than 100 mm is not recommended as explained in the companion paper. Furthermore, there are not sufficient field data to support the use of such rut depths in the design method.

**Need for Reinforcement and Need for Base Course**

If the wheel load is less than the bearing capacity of the subgrade soil, reinforcement is not needed. The bearing capacity of the subgrade soil can be calculated using Eq. (25) [derived from Eq. (23) for \( h = 0 \)] or Eq. (26) [derived from Eq. (21) for \( h = 0 \)] with \( N_c = 3.14 \).

\[ P_{h=0} = \frac{s}{f_s} \pi r^2 N_c C_{u} \]  
(25)

\[ P_{h=0} = \frac{s}{f_s} \pi r^2 N_c f_c C_{BR_{ag}} \]  
(26)

where \( f_s = 75 \text{ mm} \); \( s = \) allowable rut depth (\( \text{mm} \)); \( C_{BR_{ag}} = \) \( CBR \) of the subgrade soil; \( C_{u} = \) undrained cohesion of the subgrade soil; \( N_c = \) bearing capacity factor; \( r = \) radius of the equivalent tire contact area (\( \text{m} \)); and \( f_c = \) factor equal to 30 \( \text{ kPa} \).
for which unpaved roads are typically designed is 100,000 axle passes. For high numbers of passes, a thick base course is generally required, as shown in the design charts presented in Fig. 2. A thick base course may fail, due to lack of bearing capacity, before the subgrade soil yields. Therefore the maximum number of passes for an unpaved road with a thick base course may be the number of passes the base course can withstand. A design chart developed by Hammitt (1970) can be used to determine this maximum number of passes in the case of unreinforced bases, as explained earlier. Also, as mentioned above, if the presence of a geogrid allows for better compaction of the base course aggregate, thereby resulting in a greater bearing capacity of the base course, the number of passes a thick base course can withstand may be increased by the presence of a geogrid. However, as mentioned above, further studies would be needed to conclude on this point.

Another limitation of the method is the fact that the tensioned membrane effect is not taken into account. However, this has virtually no impact on the required base course thickness if the allowable rut depth is less than 100 mm. The tensioned membrane effect is further discussed in a subsequent section.

In the design method proposed in this study, the influence of geogrid reinforcement is accounted for in two ways: the high value of the bearing capacity factor, $N_r = 5.71$, and the aperture stability modulus, $J$. The high value of $N_r$ implies that interlock between geogrid and base course material is sufficient to generate maximum inward shear stresses at the base/subgrade interface. The aperture stability modulus has been linked to an increase in the stress distribution angle, i.e., an improvement of base course ability to distribute normal stresses. In the method presented in this study, it is assumed that all geogrids provide sufficient interlock with base course material to justify the use of $N_r = 5.71$. Therefore the only way to differentiate between various geogrids in the method presented in this study is through the aperture stability modulus, as linked to the stress distribution angle. The reality is more complex, as both the degree of interlock and the ability of the base to distribute normal stresses depend on several geogrid properties such as the thickness, stiffness, and shape of ribs, the size, shape, and rigidity of apertures, junction strength, and tensile modulus at low strains. A design method that would take all these properties into account would be difficult to develop. The method presented in this study, even though it required lengthy analyses for its development, is easy to use by designers. However, a possible limitation of the method presented in this study is that it should be used with caution in the case of geogrids that have properties that differ from those of the geogrids used in the laboratory and field tests used to calibrate the method, as these geogrids may perform significantly differently than the tested geogrids. This limitation appears in the term 0.661 in Eq. (17). This term should be greater than 0, which limits the use of the method to geogrids having an aperture stability modulus less than 0.8 mN/m. New laboratory and field tests and a new calibration of the method would be required for geogrids having an aperture stability modulus greater than 0.8 mN/m.

Comparison of Methods

In the method presented in this study, the required base course thickness for a reinforced unpaved road is calculated using a unique equation. In contrast, in the methods by Giroud and Noiray (1981) and Giroud et al. (1985), the required base course thickness for reinforced unpaved roads was determined in two steps: the first step consisted of calculating the required base course thickness for an unreinforced unpaved road on the same soil, and the second step consisted of calculating the difference between the required base course thickness for the unreinforced and the reinforced unpaved roads. The use of a unique equation is a major improvement not only from a practical standpoint, but also from a theoretical standpoint, because this unique equation is more rigorous than the earlier two-step approach.

The method presented in this study does not include the tensioned membrane effect. Giroud et al. (1985) performed systematic calculations to evaluate the tensioned membrane effect using the equations developed by Giroud and Noiray (1981). They concluded that the tensioned membrane effect is negligible if the allowable rut depth is 75 mm and reduces the required base course thickness by approximately 10% if the rut depth is 150 mm. Since the maximum allowable rut depth recommended for the safe use of the design method presented in this study is 100 mm, the tensioned membrane effect is neglected herein.

A method for the design of unpaved roads published by Steward et al. (1977) is used by several governmental agencies in the United States. This method includes two steps: (1) calculation of the normal stress at the base/subgrade interface induced by the wheel load, using the theory of elasticity for a homogeneous medium (i.e., using equations derived from Boussinesq equations); and (2) use of the classical bearing capacity equation for saturated undrained soil (i.e., $p_{r} = N_r c_u$). Essentially, the Steward et al. method consists of using Eq. (11) in the companion paper without $m$ and where $\tan \alpha$ is replaced by a stress calculation using the theory of elasticity. The Steward et al. method does not directly account for the traffic. However, two sets of values of the bearing capacity factor, $N_r$, are recommended depending on the traffic: (1) $N_r = 5.3$ for unreinforced bases and $N_r = 6.0$ for geotextile-reinforced bases subjected to a number of 80 kN axle passes less than 100; and (2) $N_r = 2.8$ for unreinforced bases and $N_r = 5.0$ for geotextile-reinforced bases subjected to a number of 80 kN axle passes greater than 1,000. Accounting for traffic through the bearing capacity factor is somehow arbitrary. The writers of this paper consider that the theoretical values of the bearing capacity factor were established on a sound theoretical basis and should not be changed to calibrate equations. However, there is no other choice when the Steward et al. method is used, as $N_r$ is the only parameter in the method. In contrast, in the method presented in this study, traffic is accounted for directly and in a way that is consistent with the general form of the relationship between base course thickness and traffic used in road design. Also, the Steward et al. method does not account for the properties of the geosynthetic and, therefore, cannot quantify the benefit that results from interlock between geogrid and base course material.

Applications

Design Procedures

The following procedures are suggested for designing unreinforced and reinforced unpaved roads.

Preliminary Step: Calculate the radius of the equivalent contact area using Eq. (2), and select the allowable rut depth (e.g., the typical value of 75 mm or another value) if it has not already been given. Also, if the $CBR$ of the subgrade soil is given and the undrained cohesion is not, the undrained cohesion should be derived from the $CBR$ using Eq. (3) or any other appropriate relationship.

Step 1: Check whether the subgrade soil itself has enough bearing capacity to support the wheel load without reinforcement.
This can be done by checking whether the wheel load, \( P \), is greater than the bearing capacity of the subgrade soil, which is given by Eq. (25) or (26) with \( N_x = 3.14 \). As indicated earlier, the symbol for the bearing capacity thus calculated is \( P_{h=0,\text{unreinforced}} \). The value of \( P_{h=0,\text{unreinforced}} \) is independent of the number of axle passes.

If the wheel load, \( P \), is less than the subgrade soil bearing capacity, \( P_{h=0,\text{unreinforced}} \), no base course is needed. However, a base course with the minimal thickness of 0.10 m is recommended for preventing disturbance of the subgrade soil due to trafficking. For \( P<P_{h=0,\text{unreinforced}} \), the design stops here. If \( P > P_{h=0,\text{unreinforced}} \), a base course, possibly with a geosynthetic, is needed. The design moves to Step 2.

Step 2: Determine the required base course thickness for unreinforced and/or reinforced roads.

The equation that gives the required base course thickness must be solved by iterations. Therefore, to determine the required base course thickness, an initial thickness must be assumed. Guidance for the selection of the initially assumed thickness is provided by the design charts presented in Fig. 2. The assumed thickness is put into any of the Eqs. (16)–(19) with other design parameters to calculate the required base course thickness. If the calculated thickness is significantly different from the assumed thickness, another iteration is needed where the thickness just calculated is used as the assumed thickness. The calculation is repeated until the calculated thickness is approximately equal to the assumed thickness. The thickness thus obtained is the required base course thickness. If the calculated thickness is less than the minimal base course thickness of 0.10 m, the minimal thickness should be used. As indicated in the discussion presented earlier, it is possible in some cases that the calculated required base course thickness be negative. In this case, the minimal base course thickness of 0.10 m should be used.

### Design Example

#### Presentation of Design Example

This design example demonstrates the use of the design method developed in this study. In it, the required base course thickness is determined for both geogrid-reinforced and unreinforced cases. The considered reinforcement is B12 geogrid; it has an aperture stability modulus of 0.65 mN/°. The unpaved road will be designed for 5,000 passes of a 40 kN wheel load with a tire pressure of 550 kPa. The subgrade soil has a CBR of 1.0 and the field base course after compaction has a CBR of 15.0. The allowable rut depth is 75 mm.

#### Solution of Design Example

The procedure presented earlier is followed. As a preliminary step, the radius of the equivalent tire contact area is calculated using Eq. (2) as follows:

\[
r = \sqrt{\frac{40}{3.14 \times 550}} = 0.152 \text{ m}
\]

The undrained cohesion of the subgrade soil is not given. The relationship between undrained cohesion and CBR, \( C_{BR} \), expressed by Eq. (3) is adopted in this design example. Therefore, in the remainder of this design example, only equations that include this relationship will be used.

The first step consists of calculating the allowable bearing capacity for the subgrade soil without reinforcement using Eq. (26) [which includes the relationship expressed by Eq. (3)] as follows, and using the symbol \( P_{h=0,\text{unreinforced}} \):

\[
P_{h=0,\text{unreinforced}} = \left( \frac{75}{75} \right) \left( \frac{\pi}{(0.152)^2} \right) \left( 3.14 \times 0.152 \times 0.50 \right) = 6.8 \text{ kN}
\]

The wheel load (40 kN) is greater than this bearing capacity. Therefore a base course with or without a geosynthetic is required. The design moves to the second step.

Prior to proceeding with the base course thickness calculation, the limited modulus ratio, \( R_E \), and the modulus ratio factor, \( f_E \), which are common to the unreinforced and the reinforced cases, are calculated.

The limited modulus ratio, \( R_E \), can be calculated using Eq. (6) as follows:

\[
R_E = \min(7.84, 5.0) = 5.0
\]

Then, Eq. (5) is used as follows to calculate the modulus ratio factor, \( f_E \):

\[
f_E = 1 + 0.20(5.0 - 1) = 1.816
\]

The calculation of the base course thickness requires iterations.

To start the iteration process for the case of the unreinforced unpaved road, it is assumed that the required base course thickness is 0.40 m. Using this assumed value of the base course thickness, the bearing capacity mobilization factor, \( m \), for the unreinforced case is calculated using Eq. (10) as follows:

\[
m = \frac{75}{75} \left[ 1 - 0.9 \exp \left( - \frac{(0.152)^2}{0.400} \right) \right] = 0.221
\]

Using Eq. (16) [which includes the relationship expressed by Eq. (3)] with \( N_x = 3.14 \) and \( J = 0 \), the first iteration of the calculation of the required thickness for the unreinforced case gives

\[
h = \frac{0.868 + (0.661)(0.152)(0.400)^{1.5}}{1.816} \log(5,000)
\]

\[
= 0.793 \times (5,140 - 1) \times 0.152 = 0.50 \text{ m}
\]

It should be noted that, in the calculations, the fraction \([40/(\pi 0.152^2)]\) represents the tire contact pressure and can be replaced by 550 kPa. The calculated base course thickness is greater than the initially assumed value of 0.40 m for the unreinforced case. The calculated value (0.50 m) is used to recalculate \( m \) and is used as the assumed value for the next iteration with Eq. (16). The process is repeated until the calculated value is approximately equal to the assumed value. The results of the iterations for the unreinforced case are listed in Table 1.

For the geogrid-reinforced case, the radius of the contact area and the modulus ratio factor are the same as those for the unreinforced case. To start the iteration process it is assumed that the

<p>| Table 1. Calculation of Base Course Thickness for Design Example Unreinforced Case |</p>
<table>
<thead>
<tr>
<th>Assumed ( h ) (m)</th>
<th>( f_E )</th>
<th>( m )</th>
<th>Calculated ( h ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>1.816</td>
<td>0.221</td>
<td>0.50</td>
</tr>
<tr>
<td>0.50</td>
<td>1.816</td>
<td>0.179</td>
<td>0.50</td>
</tr>
</tbody>
</table>


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Table 2. Calculation of Base Course Thickness for Design Example
Reinforced Case

<table>
<thead>
<tr>
<th>Assumed $h$ (m)</th>
<th>$f_E$</th>
<th>$m$</th>
<th>Calculated $h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.816</td>
<td>0.378</td>
<td>0.21</td>
</tr>
<tr>
<td>0.21</td>
<td>1.816</td>
<td>0.467</td>
<td>0.19</td>
</tr>
<tr>
<td>0.19</td>
<td>1.816</td>
<td>0.525</td>
<td>0.18</td>
</tr>
<tr>
<td>0.18</td>
<td>1.816</td>
<td>0.599</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The required base course thickness for the geogrid-reinforced case is 0.25 m. Using this assumed value of the base course thickness, the bearing capacity mobilization factor, $m$, for the geogrid-reinforced case is calculated using Eq. (10) as follows:

$$ m = \frac{75}{75} \left[1 - 0.9 \exp\left(-\frac{0.152}{0.250}\right)\right] = 0.378 $$

Using Eq. (16) with $N_c = 5.71$ and $J = 0.65$ m/N, the calculated base course thickness for the B12 geogrid-reinforced case is

$$ h = \frac{0.868 + (0.661 - 1.006 \times 0.65^2) \left(\frac{0.152}{0.250}\right)^{1.5} \log(5,000)}{1.816} \times \sqrt{\frac{40}{\pi(0.152)^2}} - \frac{0.378 \times 5.71 \times 30 \times 1.0}{0.152} = 0.706 \times (2.914 - 1)0.152 = 0.21 \text{ m} $$

The calculated base course thickness is less than the initially assumed value of 0.25 m for the reinforced case. The calculated value (0.21 m) is used to recalculate $m$ and is used as the assumed value for the next iteration with Eq. (16). The process is repeated until the calculated value is approximately equal to the assumed value. The results of the iterations for the reinforced case are listed in Table 2.

In conclusion, the required base course thickness is 0.50 m if the base is unreinforced and 0.18 m if it is reinforced with B12 geogrid. A good approximation of these two values could have been obtained using the design charts presented in Fig. 2.

Discussion of Design Example

The design example is completely solved as shown above. The calculations presented below are provided for the sole purpose of discussion. The goal of this discussion is to make the reader fully aware of the range and use of the available equations.

If the wheel load had been less than 6.8 kN, the subgrade soil bearing capacity calculated in Step 1 above, a negative value would have been obtained using any of the Eqs. (16)–(19) for both the unreinforced and the reinforced cases. The minimal base course thickness of 0.10 m would then have been selected.

If the wheel load had been less than 12.4 kN, a negative value would have been obtained using any of the Eqs. (16)–(19) for the geogrid-reinforced case. The minimal base course thickness of 0.10 m would have been selected. The value of 12.4 kN is the wheel load that can theoretically be carried by the subgrade soil just covered with a geogrid. This wheel load is calculated using Eq. (26) as follows and using the symbol $P_{h=0,GGD}$, as recommended earlier:

$$ P_{h=0,GGD} = \left(\frac{75}{75}\right) (\pi)(0.152)^2(5.71)(30)(1.0) = 12.4 \text{ kN} $$

As the wheel load (40 kN) is greater than 12.4 kN, a positive required base course thickness was obtained for both the unreinforced and the reinforced cases.

The maximum wheel load for the case of an unreinforced minimal base course thickness of 0.10 m can be calculated using any of the Eqs. (20)–(23). To perform this calculation it is necessary to first calculate the value of the bearing capacity mobilization coefficient, $m$, for a base course thickness of 0.10 m, using Eq. (10) as follows:

$$ m = \frac{75}{75} \left[1 - 0.9 \exp\left(-\frac{0.152}{0.100}\right)\right] = 0.911 $$

Then, Eq. (20) can be used as follows:

$$ P_{h=0.10} = (\pi)(0.152)^2(0.911)(3.14)(30)(1.0) \times \left[1 + \frac{0.152}{0.100}[1.816 - (0.661 - 1.006)(0.65^2)] \log 5,000\right] $$

$$ = 9.3 \text{ kN} $$

It can be checked that Eq. (16) with $P = 9.3$ kN, $N_c = 3.14$, and $J = 0$ gives $h = 0.10$ m. If the wheel load had been between 6.8 and 9.3 kN, the required base course thickness for the unreinforced case calculated using Eq. (16) with $N_c = 3.14$ would have been positive and less than 0.10 m. The minimal base course thickness of 0.10 m would then have been selected.

The maximum wheel load for the case of the minimal base course thickness of 0.10 m reinforced with geogrid B12 can be calculated using Eq. (20) as follows:

$$ P_{h=0.10} = (\pi)(0.152)^2(0.911)(5.71)(30)(1.0) \times \left[1 + \frac{0.152}{0.100}[1.816 - (0.661 - 1.006)(0.65^2)] \log 5,000\right] $$

$$ = 24.7 \text{ kN} $$

It can be checked that Eq. (16) with $P = 24.7$ kN, $N_c = 5.71$, and $J = 0.65$ m/N gives $h = 0.10$ m. If the wheel load had been between 12.4 and 24.7 kN, the required base course thickness for the geogrid-reinforced case calculated using Eq. (16) with $N_c = 5.71$ would have been positive and less than 0.10 m. The minimal base course thickness of 0.10 m would then have been selected.

Case Studies

Three wheel-loading tests (other than the Hammitt’s tests that were used to calibrate the method) were analyzed using the design method presented in this study.

Fannin and Sigurðsson’s Study

In the study by Fannin and Sigurðsson (1996), the test truck had an axle load of 80 kN (i.e., a wheel load of 40 kN) and a tire pressure of 620 kPa. Some sections were unreinforced and the other sections were reinforced with a layer of B11 geogrid at the base/subgrade interface. The base course thickness ranged between 0.25 and 0.5 m. As pointed out by Fannin and Sigurðsson...
The vane shear tests and unconsolidated undrained triaxial tests showed that the average undisturbed undrained cohesion of the subgrade soil was 40.0 kPa. However, the subgrade soil had a sensitivity of 7.0. The average remolded undrained cohesion was 5.7 kPa. The undrained cohesion of the subgrade soil during trafficking should be between undisturbed and disturbed undrained cohesions. The average undrained cohesion of 22.9 kPa is selected for the analysis herein. These undrained cohesions of 40.0, 22.9, and 5.7 kPa are equivalent to CBR values of 1.33, 0.76, and 0.19, respectively, according to Eq. (3). However, as indicated after Eq. (3), other relationships between undrained cohesion and CBR can be used.

No base course CBR value was reported. Since it was observed that a 0.5-m-thick base course failed, the chart developed by Hammitt (1970) for unsurfaced soils can be used, as pointed out earlier. This chart gives a CBR value of 10 for the base course. Using this value of \( CBR_{bc} \) and 0.76 for \( CBR_{sg} \), the relationship \( E_{bc}/E_{sg} = 3.48(CBR_{bc}^{3/4} / CBR_{sg}) \) obtained in the companion paper gives a modulus ratio of 9.1. Therefore a limited modulus ratio of 5.0 is used in the analysis, in accordance with Eq. (6).

The calculated numbers of passes for the different base course thicknesses and for the unreinforced and reinforced cases are plotted in Fig. 5. This figure shows that the calculated results compare reasonably well with the measured results, except for a base course thickness of 0.5 m, for the reason indicated above.

The data from Fannin and Sigurdsson (1996) were also used to investigate the change to CBR values of the subgrade soil during trafficking. This change could result from disturbance of the subgrade soil. The CBR values in Fig. 6 were back-calculated using the field data from Fannin and Sigurdsson (1996) and Eq. (16) or (17). Using the number of passes as input data in Eq. (16) or (17), the CBR values of the subgrade soil were determined by trial and error until the calculated rut depth matched the measured rut depth for a specific base course thickness. Fig. 6 shows that the back-calculated CBR values of subgrade soil decrease as the number of passes increases for both the unreinforced case and the geogrid-reinforced case. The reduction of CBR values of subgrade soil is attributed to increasing disturbance of the subgrade soil induced by trafficking. At a low number of passes, the back-calculated CBR values of subgrade soil are close to that of the undisturbed subgrade soil. As the number of passes increases, the back-calculated CBR values of subgrade soil approach that of the remolded subgrade soil. This interesting observation shows that, if the subgrade soil is highly sensitive, reduction of cohesion due to trafficking should be considered. However, no guidance is provided and judgment is required for determining the degree of reduction.

**Tingle and Webster’s Study**

Tingle and Webster (2003) report on results of a field test section where a 5-ton military truck with a tandem axle load of 147 kN (equivalent single wheel load= 36.8 kN) was used. The tire pressure used was 516 kPa. Four sections were tested: (1) an unreinforced section with a 0.51 m base course thickness; (2) a section with a 0.38 m base course thickness, reinforced with a nonwoven geotextile having a grab strength of 580 N; (3) a section with a 0.38 m base course thickness, reinforced with a woven geotextile having a grab strength of 1,110 N; and (4) a section with a 0.25 m base course thickness, reinforced with a B12 geogrid associated with the same nonwoven geotextile as above. All sections reached a rut depth of approximately 75 mm for approximately 2,000 vehicle passes, i.e., 4,000 axle passes.

The measured subgrade soil CBR after trafficking was 0.7. The base course material CBR after trafficking was measured to be 8.0. The modulus ratio of the base course to the subgrade calculated using the relationship \( E_{bc}/E_{sg} = 3.48(CBR_{bc}^{3/4} / CBR_{sg}) \) was 21.6. In accordance with Eq. (6), the modulus ratio was limited to 5 in the calculations performed by the writers of this paper using the method presented in this paper. The results of these calculations for 4,000 axle passes are presented in Table 3. This table shows that the base course thicknesses calculated by the design method presented in this paper for the unreinforced control section, for the two geotextile-reinforced sections, and for the geogrid/geotextile composite reinforced section are close to those used in the field tests.

Tingle and Webster (2003) used their test results to back-calculate the value of the bearing capacity factor, \( N_c \), using the Steward et al. (1977) method. Thus they obtained the following values: \( N_c = 2.6 \) for the unreinforced section, \( N_c = 3.6 \) for the geotextile reinforced section, and \( N_c = 5.8 \) for the geogrid-reinforced...
As indicated earlier, the writers of this paper consider that the theoretical values of the bearing capacity factor were established on a sound theoretical basis and should not be changed to calibrate equations. However, there is no other choice when the Steward et al. method is used, as \( N_c \) is the only parameter in the method. The good agreement between the Tingle and Webster (2003) test results and the calculations performed using the method presented in this study shows that it is not necessary to depart from the theoretical values of the bearing capacity factor when the method used to interpret the test results accounts for all relevant parameters.

### Knapton and Austin’s Study

A large laboratory test facility was utilized for moving wheel tests. The wheel load was 64 kN. The subgrade soil was imported clay with a \( CBR \) of 1.0. The thickness of the granular base was 0.4 m. The granular base was placed directly on the subgrade soil (unreinforced case section) or one layer of B12 geogrid was placed between the base and the subgrade (reinforced case).

Since no information on the tire pressure and the \( CBR \) value of the base course was included in the paper by Knapton and Austin (1996), the analysis was performed with an assumed typical tire pressure of 550 kPa and an assumed modulus ratio of the base course to the subgrade soil of 5.0. The calculated rut depth was determined by trial and error. It was varied to match the calculated number of passes with the measured number of passes for each case. The comparisons of calculated and measured rut depth with the number of passes are listed in Table 4. The comparisons show a reasonable agreement between the calculated and the measured rut depths, especially for the cases where the geogrid was used.

### Conclusions

The design method presented in this paper is based on theoretical development and is calibrated using data from field wheel load tests and laboratory cyclic plate loading tests on unreinforced and reinforced base courses, all constructed over weak subgrade soils. In contrast, the methods published previously were not calibrated using field and laboratory data. Also, in the method presented in this paper, the required base course thickness for a reinforced unpaved road is calculated using a unique equation, whereas more than one equation was needed with earlier methods.

The new design method has been shown to accurately predict the performance of unreinforced and reinforced unpaved roads measured in recent studies. Therefore the design method presented in this paper will enable designers to more accurately determine the base course thickness required to support traffic on unpaved roads, temporary construction roads, and working platforms.

### Notation

The following symbols are used in this paper:

- \( a \) = constant;
- \( b \) = constant;
- \( CBR \) = California bearing ratio;
- \( CBR_{bc} \) = base course California bearing ratio;
- \( CBR_{sg} \) = subgrade California bearing ratio;
- \( c_u \) = subgrade soil undrained cohesion;
- \( d \) = constant;

---

### Table 3. Comparison of Measured and Calculated Base Course Thickness

<table>
<thead>
<tr>
<th>Road section</th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unreinforced</td>
<td>0.51</td>
<td>0.59</td>
</tr>
<tr>
<td>Reinforced with nonwoven geotextile</td>
<td>0.38</td>
<td>0.43</td>
</tr>
<tr>
<td>Reinforced with woven geotextile</td>
<td>0.38</td>
<td>0.43</td>
</tr>
<tr>
<td>Reinforced with B12 geogrid on geotextile</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

\(^a\)Tingle and Webster (2003).
\(^b\)This study.
\[ E_{bc} = \text{base course resilient modulus}; \]
\[ E_{sg} = \text{subgrade soil resilient modulus}; \]
\[ f_c = \text{factor between undrained cohesion and } CBR \text{ of subgrade soil}; \]
\[ f_E = \text{modulus ratio factor}; \]
\[ f_S = \text{factor equal to 75 mm rut depth}; \]
\[ h = \text{thickness of base course and, generally, required thickness of base course}; \]
\[ h_{\text{min}} = \text{minimum value of the thickness of the base course}; \]
\[ h^* = \text{normalized base course thickness}; \]
\[ J = \text{aperture stability modulus of geogrid}; \]
\[ m = \text{bearing capacity mobilization coefficient}; \]
\[ N = \text{number of passes of axle}; \]
\[ N_c = \text{bearing capacity factor}; \]
\[ n = \text{constant}; \]
\[ P = \text{load applied by one of the wheels in the case of single-wheel axles and the load applied by a set of two wheels in the case of dual-wheel axles}; \]
\[ P_{h=0} = \text{wheel load that can be carried without a base course}; \]
\[ P_{h=0, GGD} = \text{wheel load that can be carried without a base course but with geogrid reinforcement on top of the subgrade soil}; \]
\[ P_{h=0, GTX} = \text{wheel load that can be carried without a base course but with geotextile reinforcement on top of the subgrade soil}; \]
\[ P_{h=0, \text{unreinforced}} = \text{wheel load that can be carried without a base course and without reinforcement, i.e., bearing capacity of the subgrade soil}; \]
\[ p = \text{tire contact pressure}; \]
\[ R_E = \text{limited modulus ratio of base course to subgrade soil}; \]
\[ r = \text{radius of equivalent tire contact area}; \]
\[ s = \text{rut depth and, generally, allowable rut depth}; \]
\[ \xi = \text{constant}; \]
\[ \omega = \text{constant}. \]

References


